Paper Reference(s)

6681/01

Edexcel GCE

Mechanics M5

Advanced Level

Friday 12 June 2015 – Morning

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M5), the paper reference (6681), your surname, other name and signature.
Whenever a numerical value of g is required, take $g = 9.8 \text{ m s}^{-2}$.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 7 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

1. A particle P moves from the point A, with position vector $(2\mathbf{i} + 4\mathbf{j} + a\mathbf{k})$ m, where a is a positive constant, to the point B, with position vector $(-\mathbf{i} + a\mathbf{j} - \mathbf{k})$ m, under the action of a constant force $\mathbf{F} = (2\mathbf{i} + a\mathbf{j} - 3\mathbf{k})$ N. The work done by $\mathbf{F}$, as it moves the particle P from A to B, is 3 J. Find the value of a.

(6)

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2. A particle P moves so that its position vector, $\mathbf{r}$ metres, at time t seconds, where $0 \leq t < \frac{\pi}{2}$, satisfies the differential equation

$$\frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t} - (\tan t)\mathbf{r} = (\sin t)\mathbf{i}.$$

When $t = 0$, $\mathbf{r} = -\frac{1}{2}\mathbf{i}$.

Find $\mathbf{r}$ in terms of t.

(8)

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3. A rigid body is in equilibrium under the action of three forces $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$.

$\mathbf{F}_1$ and $\mathbf{F}_2$ act at the points with position vectors $\mathbf{r}_1$ and $\mathbf{r}_2$ respectively, where

$\mathbf{F}_1 = (2\mathbf{j} + \mathbf{k})$ N, $\quad \mathbf{r}_1 = (\mathbf{i} + 2\mathbf{j} + 2\mathbf{k})$ m

$\mathbf{F}_2 = (-2\mathbf{i} - \mathbf{j})$ N, $\quad \mathbf{r}_2 = (-\mathbf{i} - \mathbf{j} + \mathbf{k})$ m.

(*a*) Find the magnitude of $\mathbf{F}_3$.

(4)

(*b*) Find a vector equation of the line of action of $\mathbf{F}_3$, giving your answer in the form $\mathbf{r} = \mathbf{a} + t\mathbf{b}$, where $\mathbf{a}$ and $\mathbf{b}$ are constant vectors and t is a scalar parameter.

(8)

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4. A particle P, whose initial mass is m_0, is projected vertically upwards from the ground at time $t = 0$ with speed $\frac{g}{k}$, where k is a constant. As the particle moves upwards it gains mass by picking up small droplets of moisture from the atmosphere. The droplets are at rest before they are picked up. At time t the speed of P is v and its mass has increased to $m_0\mathrm{e}^{kt}$. Assuming that, during the motion, the acceleration due to gravity is constant,

(*a*) show that, while P is moving upwards,

$$kv + \frac{\mathrm{d}v}{\mathrm{d}t} = -g,$$

(6)

(*b*) find, in terms of m_0, the mass of P when it reaches its greatest height above the ground.

(6)

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5. A uniform circular disc, of mass m and radius a, is free to rotate about a fixed smooth horizontal axis L. The axis L is a tangent to the disc at the point A. The centre O of the disc moves in a vertical plane that is perpendicular to L.

The disc is held at rest with its plane horizontal and released.

(*a*) Find the angular acceleration of the disc when it has turned through an angle of $\frac{\pi}{3}$.

(5)

(*b*) Find the magnitude of the component, in a direction perpendicular to the disc, of the force of the axis L acting on the disc at A, when the disc has turned through an angle of $\frac{\pi}{3}$.

(4)

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6. A pendulum is modelled as a uniform rod AB, of mass $3m$ and length $2a$, which has a particle of mass $2m$ attached at B. The pendulum is free to rotate in a vertical plane about a fixed smooth horizontal axis L which passes through A. The vertical plane is perpendicular to the axis L.

(*a*) Find the period of small oscillations of the pendulum about its position of stable equilibrium.

(8)

The pendulum is hanging at rest in a vertical position, with B below A, when it is given a horizontal impulse of magnitude J. The impulse acts at B in a vertical plane which is perpendicular to the axis L.

Given that the pendulum turns through an angle of 60° before first coming to instantaneous rest,

(*b*) find J.

(8)

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7. (*a*) Find, using integration, the moment of inertia of a uniform solid hemisphere, of mass m and radius a, about a diameter of its plane face.

[*You may assume, without proof, that the moment of inertia of a uniform circular disc, of mass m and radius r, about a diameter is* $\frac{1}{4}mr^2$.]

(10)

(*b*) Hence find the moment of inertia of a uniform solid sphere, of mass M and radius a, about a diameter.

(2)

TOTAL FOR PAPER: 75 MARKS

END

Question Number	Scheme	Marks
1.	$\mathbf{AB} = (-3\mathbf{i} + (a-4)\mathbf{j} + (-1 - a)\mathbf{k})$ Work done = 3 = $(2\mathbf{i} + a\mathbf{j} - 3\mathbf{k}).(-3\mathbf{i} + (a-4)\mathbf{j} + (-1-a)\mathbf{k})$ $3 = -6 + a(a-4) - 3(-1-a)$ $0 = a^2 - a - 6$ $0 = (a-3)(a+2)$ $a = 3$ since $a > 0$.	B1 M1 A1 A1 M1 A1 **6**
	Notes	
	B1 for correct **AB** in any form. First M1 for 3 = (2**i** + a**j** - 3**k**).their **AB** (allow **BA**) Need an attempt. First A1 for any correct equation Second A1 for $0 = a^2 - a - 6$ Second M1 for solving a quadratic (2 solutions) Third A1 for $a = 3$	

Question Number	Scheme	Marks
2	$\text{IF} = \mathrm{e}^{\int -\tan t\,\mathrm{d}t} = \cos t$	M1 A1
	$\frac{\mathrm{d}}{\mathrm{d}t}(\mathbf{r}\cos t) = \sin t \cos t\,\mathbf{i}$	
	$\mathbf{r}\cos t = \int \sin t \cos t\,\mathbf{i}\,\mathrm{d}t$	M1
	$\mathbf{r}\cos t = \frac{1}{2}\sin^2 t\,\mathbf{i}\,(+\mathbf{C})$	A1
	$t = 0, \mathbf{r} = -\frac{1}{2}\mathbf{i} \Rightarrow \mathbf{C} = -\frac{1}{2}\mathbf{i}$	M1 A1
	$\mathbf{r}\cos t = \frac{1}{2}\sin^2 t\,\mathbf{i} - \frac{1}{2}\mathbf{i}$	
	$\mathbf{r} = -\frac{1}{2}\cos t\,\mathbf{i}$ oe	**DM1** A1
		8
	Notes	
	First M1 for $\mathrm{e}^{\int -\tan t\,dt}$ (allow if – sign omitted) First A1 for $\cos t$ Second M1 see scheme (multiply <u>both</u> sides by IF and integrate) Second A1 for a correct equation (without **C**) $(-\frac{1}{2}\cos^2 t$ or $-\frac{1}{4}\cos 2t)$ Third M1 for use of initial conditions Third A1 for a correct **C** Fourth M1 **dependent** on second M1 for producing an expression for **r** Fourth A1 for **r =** any equivalent form (does not need to be simplified)	

Question Number	Scheme	Marks
3(a)	$(2\mathbf{j}+\mathbf{k}) + (-2\mathbf{i}-\mathbf{j}) + \mathbf{F}_3 = \mathbf{0}$ => $\mathbf{F}_3 = (2\mathbf{i}-\mathbf{j}-\mathbf{k})$ N Magnitude $= \sqrt{2^2+(-1)^2+(-1)^2} = \sqrt{6}$ N	M1 A1 M1 A1 (4)
3(b)	$(\mathbf{i}+2\mathbf{j}+2\mathbf{k}) \times (2\mathbf{j}+\mathbf{k}) + (-\mathbf{i}-\mathbf{j}+\mathbf{k}) \times (-2\mathbf{i}-\mathbf{j}) + (x\mathbf{i}+y\mathbf{j}+z\mathbf{k}) \times (2\mathbf{i}-\mathbf{j}-\mathbf{k})$ $= -2\mathbf{i}-\mathbf{j}+2\mathbf{k} + \mathbf{i}-2\mathbf{j}-\mathbf{k} + (-y+z)\mathbf{i}+(2z+x)\mathbf{j}+(-x-2y)\mathbf{k}$ $-1-y+z=0$ $-3+2z+x=0$ $1-x-2y=0$ $x=1, y=0, z=1$ is a solution $\mathbf{r} = (\mathbf{i}+\mathbf{k}) + t(2\mathbf{i}-\mathbf{j}-\mathbf{k})$	M1 A3 M1 A1 M1 A1 (8) **12**
3(b) Alt Concurrency Principle	$\mathbf{r} = (\mathbf{i}+2\mathbf{j}+2\mathbf{k}) + \lambda(2\mathbf{j}+\mathbf{k})$ $\mathbf{r} = (-\mathbf{i}-\mathbf{j}+\mathbf{k}) + \mu(-2\mathbf{i}-\mathbf{j})$ $1 + 0 = -1-2\mu$ $2+2\lambda = -1-\mu$ $2+\lambda = 1$ => $\lambda = \mu = -1$ so point has pv $(\mathbf{i}+\mathbf{k})$ $\mathbf{r} = (\mathbf{i}+\mathbf{k}) + t(2\mathbf{i}-\mathbf{j}-\mathbf{k})$	M1 A3 M1 A1 M1 A1
	Notes	
3(a)	First M1 for $\Sigma\mathbf{F}_i = \mathbf{0}$ First A1 for $\mathbf{F}_3 = (2\mathbf{i}-\mathbf{j}-\mathbf{k})$ Second M1 for $\lvert\mathbf{F}_3\rvert = \sqrt{2^2+(-1)^2+(-1)^2}$ Second A1 for $\sqrt{6}$ or 2 sf or better.	
3(b)	First M1 for consistent $\Sigma\, \mathbf{r} \times \mathbf{F}$ or $\Sigma\, \mathbf{F} \times \mathbf{r}$ using their $\mathbf{F}_3$ First A3 for correct vector products (for either of above) -1 for each incorrect product Second M1 for equating all 3 components to zero Fourth A1 for 3 correct equations Third M1 for trying to find a point and getting an equation in correct form (or any other complete method) Fifth A1 for answer (non-unique) **N.B.** They could take moments about another point e.g. $\mathbf{r}_1$ or $\mathbf{r}_2$	
3(b) Alt	First M1 for finding equations of lines of action (and later equating) First A3 for correct equations -1 each error Second M1 for equating all 3 components Fourth A1 for 3 correct equations Third M1 for trying to find a point and getting an equation in correct form (or any other complete method) Fifth A1 for answer (non-unique)	

Question Number	Scheme	Marks
4(a)	$(m+\delta m)(v+\delta v)-mv=-mg\delta t$	M1 A1
	$m\delta v+v\delta m=-mg\delta t$	**DM1**
	$\dfrac{\mathrm{d}v}{\mathrm{d}t}+\dfrac{v}{m}\dfrac{\mathrm{d}m}{\mathrm{d}t}=-g$	
	$m=m_0e^{kt} \Rightarrow \dfrac{\mathrm{d}m}{\mathrm{d}t}=m_0ke^{kt}(=km)$	M1 A1
	$\dfrac{\mathrm{d}v}{\mathrm{d}t}+\dfrac{v}{m}km=-g$ i.e. $kv+\dfrac{\mathrm{d}v}{\mathrm{d}t}=-g$ PRINTED	A1 (6)
(b)	$\displaystyle\int_{\frac{g}{k}}^{v}\frac{\mathrm{d}v}{kv+g}=-\int_0^T \mathrm{d}t$	M1
	$\dfrac{1}{k}\left[\ln(kv+g)\right]_v^{\frac{g}{k}}=T$	M1 A1
	$v=0 \Rightarrow \dfrac{1}{k}\ln 2=T$	**DM1**
	$m=m_0e^{kT}=2m_0$	M1 A1
		(6)
		12
4(b)	**OR** $\displaystyle\int\frac{\mathrm{d}v}{g+kv}=-\int \mathrm{d}t$	M1
	$\dfrac{1}{k}\ln(g+kv)=-t+(C)$	A1
	$t=0, v=\dfrac{g}{k} \Rightarrow C=(\dfrac{1}{k}\ln 2g)$	M1
	$\dfrac{1}{k}\ln(g+kv)=-t+\dfrac{1}{k}\ln 2g$	
	Put $v=0,\ t=\dfrac{1}{k}\ln 2$	**DM1**
	$m=m_0\mathrm{e}^{kt}=2m_0$	M1 A1 (6)

4(b)	$\frac{\mathrm{d}v}{\mathrm{d}t} + kv = -g$ $\text{IF} = \mathrm{e}^{kt}$ $\frac{\mathrm{d}(v\mathrm{e}^{kt})}{\mathrm{d}t} = -g\mathrm{e}^{kt}$ $v\mathrm{e}^{kt} = \int -g\mathrm{e}^{kt}\,\mathrm{d}t$ **OR** $v\mathrm{e}^{kt} = -\frac{g}{k}\mathrm{e}^{kt} + C$ $t = 0, v = \frac{g}{k} \Rightarrow C = (\frac{2g}{k})$ $v\mathrm{e}^{kt} = -\frac{g}{k}\mathrm{e}^{kt} + \frac{2g}{k}$ Put $v = 0,\ \mathrm{e}^{kt} = 2$ $m = m_0\mathrm{e}^{kt} = 2m_0$	 M1 A1 M1 **DM1** M1 A1 (6)
	Notes	
4(a)	First M1 for momentum equation (correct number of terms, excluding any $\delta m \delta v$ terms) First A1 for a correct equation Second M1, dependent on first M1, for simplifying and dividing by δt and taking limits Third M1 for differentiating the mass equation Second A1 for $\frac{\mathrm{d}m}{\mathrm{d}t} = km$ Third A1 for PRINTED ANSWER	
4(b)	First M1 for separating and integrating First A1 correct equation (without C) Second M1 for using limits or conditions Third M1, **dependent on first M1**, for putting $v = 0$ to give an equation in k and t only Fourth M1 for solving for m Second A1 for correct answer N.B. If they put v = 0 in DE and use dv/dt = -g, NO MARKS	

Question Number	Scheme	Marks
5(a)	$I_T = \frac{1}{4}ma^2 + ma^2$ $= \frac{5}{4}ma^2$ $mga\cos\frac{\pi}{3} = \frac{5}{4}ma^2\ddot{\theta}$ $\frac{2g}{5a} = \ddot{\theta}$	M1 A1 M1 **A1ft** A1 (5)
(b)	$mg\cos\frac{\pi}{3} \pm X = ma\ddot{\theta}$ $\lvert X \rvert = \frac{1}{2}mg - \frac{2}{5}mg$ $= \frac{1}{10}mg$	M1 A1 A1 A1 (4) **9**
	Notes	
5(a)	First M1 for use of perp <u>and</u> parallel axes theorem First A1 $5ma^2/4$ Second M1 for moments about the axis or differentiate a general energy equation Second A1ft on their I for correct equation Third A1 for answer	
5(b)	First M1 for resolving along the rod First A1 A1 for a correct equation (A1 for each side) Need $\pi/3$ Third A1 for $mg/10$ (<u>must be positive</u>)	

Question Number	Scheme	Marks
6(a)	$I_L = \frac{1}{3}3m(2a)^2 + 2m(2a)^2 = 12ma^2$	M1 A1
	$M(L), \quad -3mga\sin\theta - 2mg.2a\sin\theta = 12ma^2\ddot{\theta}$	M1 **A2 ft**
	$-\dfrac{7g\sin\theta}{12a} = \ddot{\theta}$	
	For small θ, $\sin\theta \approx \theta$, $\quad -\dfrac{7g\theta}{12a} = \ddot{\theta}$ so SHM with $\omega = \sqrt{\dfrac{7g}{12a}}$	M1
	$so, \quad T = \dfrac{2\pi}{\omega} = 2\pi\sqrt{\dfrac{12a}{7g}}$	M1 A1 (8)
(b)	$3mga(1-\cos 60^\circ) + 2mg2a(1-\cos 60^\circ) = \frac{1}{2}12ma^2\omega^2$	M1 A1 **A1ft**
		A1 (3rd **DM**)
	$J.2a = 12ma^2\omega$	M1 A1 **A1f**t
	$J = 6ma\sqrt{\dfrac{7g}{12a}} = 6m\sqrt{\dfrac{7ga}{12}} = m\sqrt{21ag}$	A1 (8) **16**
	Notes	
6(a)	First M1 for finding MI First A1 for $12ma^2$ Second M1 for moments about the axis Second and Third A1 **ft**, on their *I*, for correct equation (A1 for each side) They may use CM of rod + particle Third M1 for small angle approx. and comparison with standard SHM to give an ω value (need – sign in their DE) Fourth M1 for use of $2\pi / \omega$ Fourth A1 cao for any equivalent answer	
6(b)	First M1 for energy equation First A1 **ft** for KE term Second A1 for PE terms Third A1 is now 3rd **DM** mark, dependent on both previous M marks, for solving for *J* and precedes final A mark Second M1 for imp-momentum equation Fourth A1 for LHS on scheme Fifth A1 **ft** for RHS on scheme Sixth A1 cao for any equivalent answer	

<table>
<tr><th>Question Number</th><th>Scheme</th><th>Marks</th></tr>
<tr><td>7(a)</td><td>

$$\delta V = \pi y^2 \delta x$$

$$\delta m = \pi y^2 \delta x \frac{3m}{2\pi a^3}$$

$$\delta I = \tfrac{1}{4}\delta m y^2 + \delta m x^2$$

$$= \tfrac{1}{4}\delta m (y^2 + 4x^2)$$

$$= \tfrac{1}{4}\pi(a^2 - x^2)\delta x \frac{3m}{2\pi a^3}(a^2 - x^2 + 4x^2)$$

$$= \tfrac{1}{4}\pi(a^2 - x^2)(a^2 + 3x^2)\delta x \frac{3m}{2\pi a^3}$$

$$= \frac{3m}{8a^3}(a^4 + 2a^2x^2 - 3x^4)\delta x$$

$$I = \frac{3m}{8a^3}\int_0^a (a^4 + 2a^2x^2 - 3x^4)\,\mathrm{d}x$$

$$= \frac{3m}{8a^3}\left[a^4x + \frac{2a^2x^3}{3} - \frac{3x^5}{5}\right]_0^a$$

$$= \frac{2ma^2}{5}$$

</td><td>M1
M1
M1 M1 A1
M1
A1
M1
A1
A1 (10)</td></tr>
<tr><td>(b)</td><td>

$$I = 2 \text{ x } 2 \text{ x } (\frac{1}{2}M)\frac{a^2}{5}$$

$$= \frac{2Ma^2}{5}$$

</td><td>M1
A1 (2)
12</td></tr>
<tr><td></td><td>Notes</td><td></td></tr>
<tr><td>7(a)</td><td>First M1 for vol. element
Second M1 for their δV x correct density
Third M1 for ¼ $\delta m y^2$
Fourth M1 for use of parallel axes
First A1 for a correct expression in terms of x, y and δm
Fifth M1 for sub for δm and y
Second A1 for a correct δI in terms of x only
Sixth M1 for integrating with correct limits
Second A1 for correct integral
Third A1 for the answer</td><td></td></tr>
<tr><td>7(b)</td><td>M1 for use of additive rule <u>with adjusted mass</u>
A1 for correct answer
N.B. No marks for non-hence method</td><td></td></tr>
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